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The Production of Soft Secondaries by Mesotrons

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I. Cosmic-ray evidence shows that the soft component, which persists even under great thicknesses of matter, consists, primarily at least, of electrons and γ-rays. The observations of the magnitude and variation with depth of the soft component and showers, and of the size and material dependence of bursts, lead to the following rough phenomenological description: In addition to ordinary ionization losses, the mesotrons have an appreciable chance of transferring a considerable fraction of their energy to the soft component. For transfers above 2×10^{10} v this probability is roughly independent of mesotron energy. Under 10^{10} v the probability of large transfers is much greater, roughly 20 times as great as at higher energies. II. The production of secondaries of energy $<10^{10}$ v can be accounted for by a familiar process: the Lorentz contracted Coulomb field of the mesotron, in sweeping over an atom, ejects a high energy electron. This mechanism, however, is inadequate to explain the very large energy transfers, $>10^{10}$ v, involved in bursts. The theory which has been developed to describe the mesotron of spin one associates with transitions in which the direction of the mesotron spin changes an intrinsic dipole moment; this dipole field favors high energy transfers, and, roughly, can account for the observed bursts. However, the dipole field also gives probabilities of high energy bremsstrahlung, $>10^{10}$ v, much too large to be compatible with the observations. III. The cross sections for the production of electron secondaries and bremsstrahlung by the intrinsic mesotron dipole field have been estimated under the assumption that the coupling of mesotron and electromagnetic field is small; when this is not so the formula cannot be right. Examination of the approximation made indicates that the estimate of the cross section for electron secondaries should be valid up to mesotron energies of $\sim10^{12}$ v, and is thus applicable to the bursts. The bremsstrahlung formula, on the other hand, fails at 10^{10} v; it thus cannot be used for bursts, while for energies $10^9<E<10^{10}$ it leads to no contradiction with the experimental evidence. The problem of extending the formulae above these critical energies probably goes beyond the framework of present theory. The evidence indicates that it is the largeness of the coupling, and not the occurrence of lengths smaller than the critical $\hbar/\mu c\sim2\times10^{-13}$ cm, that limits the applicability of the quantum mechanics.

I. INTRODUCTION: OBSERVATIONS

THE persistence of the soft component of cosmic radiation under great thicknesses of matter shows that this is secondary radiation produced by the mesotrons of the penetrating component. At sea level the cascade radiation which comprises the soft component still has three sources of roughly equal importance:[1] degraded primary cascade radiation of high initial energy; secondaries produced by the mesotrons in matter; and (less certainly) secondaries from the disintegration of mesotrons in the atmosphere. But after a few meters water equivalent below sea level only the second group of secondaries is of importance.

From this point on, the fraction of soft com-

[1] We are grateful to Professor E. Amaldi for informing us of unpublished experimental results. For a general discussion, see Euler and Heisenberg, Ergeb. d. exakt. Naturwiss. **17**, 1 (1938).

ponent increases with depth.[2] Of the order of 10 percent of the ionizing particles are soft at 10 m under sea level; the proportion is about doubled at 30 m. The number of showers and of bursts also tails off less rapidly with filtration than the number of mesotrons. At 75 m below sea level the relative number of showers is about four times that at 10 m. This soft radiation has the typical behavior of cascade radiation of electrons and γ-rays, and there is no evidence that heavy particles or low energy mesotrons play an important part in it.

These results may be simply interpreted: filtration increases the mean mesotron energy, and for a power law energy distribution this increase is linear with depth. Thus the increase in the soft component with filtration means that the mean energy transfer from mesotrons to soft component must increase with mesotron energy.

Some information on the energy transfer cross section in the region of very high energies is given by what is known of the variation of burst frequency with burst size and with material. A most striking result here[3] is that the distribution of bursts in size, above roughly 10^{10} v, follows the same law as the energy distribution of the penetrating mesotrons: the number of bursts greater than S in size is given

$$N_{>S}\sim S^{-\gamma},\quad \gamma\sim 1.8, \tag{1}$$

by whereas the number of mesotrons of energy greater than E is given by the law, obtained from the absorption curve on the assumption that most of the absorption comes from ionization losses,

$$N_{>E}\sim E^{-\gamma},\quad \gamma\sim 1.9. \tag{2}$$

This parallelism of the burst frequency and absorption curves follows immediately if for high energies the probability of a given fractional transfer of energy from a mesotron to soft radiation is independent of mesotron energy. Such a law of energy transfer does, in fact, give a mean energy loss just proportional to the mesotron energy.

Additional evidence on the form of the cross section is provided by observations on the relative burst frequencies in different materials. As is discussed in more detail elsewhere,[4] a cross section depending only on fractional energy loss gives, for the frequency of bursts in substances of different atomic number, a Z dependence

$$(g(Z)/Z)I(Z)^{1-\gamma}, \tag{3}$$

where $g(Z)$ gives the atomic number dependence per atom of the large energy transfers, and $I(Z)$ is the critical energy of the cascade theory, which increases with decreasing Z. It seems probable that, by taking into account the effect of scattering on the low energy cascade radiation, this result, if $g(Z)=Z$, can be reconciled with the observed approximate equality of burst frequencies in different materials.

These considerations suggest that, in addition to the ordinary ionization losses, mesotrons can transfer to the soft component an appreciable fraction of their total energy, and that the probability of this is roughly independent of the mesotron energy. It is, however, not possible to fix the constant determining this probability to give agreement both with the increase in soft radiation on filtration and with the absolute value for the probability that a high energy mesotron make a burst. To see this, let us write for the cross section per atom for a mesotron of energy E and mass μ to transfer an energy fE to the soft component

$$d\sigma=\sigma_0\kappa(f)df,\quad \sigma_0=\pi Ze^4/\mu^2c^4. \tag{4}$$

Then the probability that in a centimeter of material a mesotron of energy greater than E makes a burst of energy greater than E is just

$$\xi=\sigma_0 N\int_0^1 \kappa(f)f^\gamma df, \tag{5}$$

where N is the number of atoms per cc of the material. According to Schein and Gill[5] the probability that a mesotron will make a burst of energy greater than 2×10^{10} v is about $\xi=2\times 10^{-5}$ per cm.

An independent check on this figure we may obtain by the following argument: the maximum of the transition curve for bursts in Pb, at $\sim$5 cm, is about twice the value at greater thick-

[2] W. M. Nielsen and K. Z. Morgan, Phys. Rev. **54**, 245 (1938); P. Auger and T. Grivet, Rev. Mod. Phys. **11**, 232 (1939).

[3] A. Sittkus, Zeits. f. Physik **112**, 626 (1939); M. Schein and P. S. Gill, Phys. Rev. **55**, 1111 (1939).

[4] J. R. Oppenheimer, Rev. Mod. Phys. **11**, 264 (1939).

[5] Schein and Gill, reference 3.

nesses. Thus the bursts produced by cascade radiation give a contribution equal to that from the mesotrons: decay electrons will not give an appreciable contribution. The total number of cascade electrons is about 1/20 the number of mesotrons. Of these the theory of showers shows that $(4\times 10^7/\epsilon)^{1.9}$ have an energy greater than ϵ. On the other hand, if $\epsilon > 10^{10}$, roughly $(2\times 10^9/\epsilon)^{1.9}$ mesotrons have an energy greater than ϵ. The bursts they make have a "range" of about 5 cm in Pb. Thus we must have, for the probability ξ that a mesotron of energy $>\epsilon$ make a burst in a cm of Pb,

$$5\xi(2\times 10^9/\epsilon)^{1.9} = (1/20)(4\times 10^7/\epsilon)^{1.9},$$
$$\xi = (1/100)(1/50)^{1.9} \sim 0.5\times 10^{-5}.$$

The value $\xi\sim 10^{-5}$, gives

$$\kappa = \int_0^1 \kappa(f) f^\gamma df \sim \tfrac{1}{2}. \qquad (6)$$

Now this value is not great enough to account for the increase in relative soft radiation with filtration. To see this we have to compare the total energy lost to the soft component per cm by (4) with the mean ionization energy loss of the secondaries, which gives then directly the number of soft particles per mesotron. This ratio is

$$R = m\bar{E}\int_0^1 \kappa(f) f df / 4\mu^2 c^2 \ln\,(I(Z)/ZRh). \qquad (7)$$

Here m is the electron mass and $\bar{E}$ the mean mesotron energy. This gives, for the increase in ratio of soft to hard component on filtration by 30 m water equivalent, if we use (6) to estimate κ,[6] only 0.5 percent, whereas the experimental value is more nearly 10 percent.

Since the mesotron energies and energy transfers involved in the increase of the soft component are primarily less than 10^{10} v, and the bursts involve energies greater than 2×10^{10} v, we are led to the following rough phenomenological description: In addition to ordinary ionization losses, the mesotrons have an appreciable chance of transferring a considerable fraction of their energy to the soft component. For transfers above 2×10^{10} v this probability is roughly independent of mesotron energy and is given by (4) and (6). Under 10^{10} v the probability of large transfers is much greater, and in this range it is roughly 20 times as great as at higher energies.

II. Electron Secondaries and Bremsstrahlung

The elementary processes by which mesotrons transfer energy to the soft component may conveniently be classified as nuclear or electromagnetic, according to whether the coupling of mesotrons with the heavy particles or with the electromagnetic field plays the primary part in them. Because of the large nuclear coupling and high mesotron mass it has usually been assumed that the nuclear processes would alone be important for high energy transfers. In fact, calculations of nuclear effects, such as mesotron absorption and scattering by nuclei, calculations based on the perturbation theoretic treatment of the coupling as small, lead to cross sections so large, for high energies, that they completely contradict the high penetrating power of the mesotrons. It has been emphasized especially by Heisenberg[7] that the prediction of these large cross sections rests on the essentially erroneous treatment of the interactions as small; despite several attempts no reliable estimate of them has been given, and this problem probably goes beyond the framework of present theory. Nor is it sure in what way such impacts will give appreciable amounts of soft radiation: the emission of γ-rays from nuclei excited by such impacts hardly seems a very plausible mechanism; the radiative capture of a mesotron by a nucleus, inverse to the mesotron photo-effect, seems more probable.

Under these circumstances we have thought it profitable to re-examine the electromagnetic effects a little more closely. For the problem of energy transfers to the soft component two types of collision are of first importance: elastic impacts with free electrons, and bremsstrahlung: to this latter corresponds, for the inverse problem of the creation of mesotrons by the soft component, pair production.

[6] We have supposed $\int^1 \kappa(f) f^\gamma df \sim \int_0^1 \kappa(f) f df$; for a distribution $\kappa(f)$ which greatly favored the emission of slow electrons this would no longer be right. For example $\kappa(f)\sim 1/f^2$ would increase the fraction of soft component of energy $>E_{\min}$ by a factor $\ln\,(\bar{E}/E_{\min})$.

[7] W. Heisenberg, Zeits. f. Physik **113**, 61 (1939).

If mesotrons satisfied the Dirac equation, cross sections for these two processes could be obtained from the familiar calculations for electrons. For the cross section per atom that a mesotron of energy E gives a fraction f of its energy to an electron,

$$d\sigma=\frac{2\pi Ze^4}{mc^2E}\frac{df}{f^2},\quad f<1\Big/\left(1+\frac{\mu^2c^2}{2mE}\right)$$
$$=2\sigma_0(\mu^2c^2/mE)df/f^2. \tag{8}$$

Similarly, for the cross section per atom that a γ-ray of energy fE be emitted

$$d\sigma=\frac{4\alpha Z\sigma_0}{\pi}\ln\left[\frac{2E(1-f)}{\mu c^2Z^{\frac{1}{3}}f}\right]\left(f+\frac{4}{3}\frac{(1-f)}{f}\right)df, \tag{9}$$
$$\alpha=e^2/\hbar c.$$

In this result atomic screening has been neglected, but the Coulomb field has been smoothed out for $r<Z^{\frac{1}{3}}\hbar/\mu c$; and the Born approximation has, of course, been used.

The cross section (8) gives a logarithmic increase of the soft radiation with filtration; the fraction of the electrons of energy greater than $E_{\min}$ at depth t is

$$R=\tfrac{1}{2}\ln\left(\frac{2m\epsilon^2}{\mu^2c^2E_{\min}}\right)\Big/\ln\left(\frac{I(Z)}{ZRh}\right)$$
$$\sim\frac{\ln 10(\epsilon/10^9)^2}{36-4\ln Z}\quad\text{for}\quad E_{\min}=10^7\text{ v}, \tag{10}$$

where ϵ is the ionization energy loss of the mesotron in penetrating to the depth t. This gives, at sea level, $R=13$ percent, at 30 m below sea level, with $Z=12$, $R=25$ percent, at 80 m $R=31$ percent, in rough agreement with the observed values, although perhaps somewhat low at 80 m. From (9) we obtain $R\sim3\times10^{-5}Z(\epsilon/10^9)$, a much smaller contribution than (10).

For showers of energies $\times10^9$ v $<E<10^{10}$ v, (8) gives, instead of (1), the law $N_{>S}\sim S^{-\gamma-1}$. Both the size dependence and absolute number of showers predicted by (8) agree well with the experimental results.*

However, (8) has the wrong energy dependence, (9) the wrong Z dependence, to account for the observations on bursts. But it is interesting to note that both (8) and (9) give for bursts $>2\times10^{10}$ v, a probability in rough agreement with observation; thus (9) gives

$$\kappa\sim3\alpha Z\sim2\text{ for Pb},$$

whereas the experimental value is $\kappa\sim\frac{1}{2}$. Also (8) gives, for all Z and bursts $>2\times10^{10}$, a burst frequency corresponding to $\kappa\sim\frac{1}{2}$.

There is, however, valid ground for doubting the applicability to the burst processes of (8) and (9). For Yukawa's theory requires an integral mesotron spin, and the spin dependence of nuclear forces shows that this spin must be one. The theory which has been developed to describe these particles[8] makes the mesotron electromagnetic current density depend on derivatives of the mesotron field, and associates, with transitions in which the direction of the mesotron spin changes, current distributions more singular than those of a Dirac electron. The Fourier components of current corresponding to a momentum $\hbar k>\mu c$ behave like $k^{\frac{1}{2}}$; for the Dirac electron, and for mesotron transitions not changing the spin, they behave like $k^{-\frac{1}{2}}$. For a classical point charge they behave like k^0, and for a point dipole like k; thus one may roughly ascribe these singular currents to an intrinsic mesotron dipole moment. The Fourier components of current corresponding to this dipole moment are larger by a factor $k/\mu c$ than those corresponding to the electric charge.

These singular currents of course interact very strongly with high frequency radiation fields. They radically alter the high energy cross sections, giving much larger values than (8) and (9). At the same time they introduce couplings so large that the question of the perturbation theoretic estimate of the cross section requires re-examination.

This behavior of the cross sections is well illustrated by the work of Laporte[9] on the Coulomb scattering of mesotrons. Treating this problem by the Born approximation, one finds for scattering without spin change the Rutherford result

$$d\sigma=\tfrac{1}{2}Z\sigma_0(\mu c^2/E)^2\csc^4\tfrac{1}{2}\theta d(\cos\theta),\quad E\gg\mu c^2. \tag{11}$$

* See A. C. B. Lovell, Proc. Roy. Soc. **172**, 583 (1939).

[8] Yukawa, Sakata, and Taketani, Proc. Phys.-Math. Soc. Japan **20**, 319 (1938); Yukawa, Sakata, Kobayasi, and Taketani, Proc. Phys.-Math. Soc. Japan **20**, 720 (1938); N. Kemmer, Proc. Roy. Soc. **A166**, 127 (1938); Fröhlich, Heitler, and Kemmer, Proc. Roy. Soc. **A166**, 154 (1938); H. J. Bhabha, Proc. Roy. Soc. **A166**, 501 (1938).

[9] O. Laporte, Phys. Rev. **54**, 905 (1938).

For scattering with spin change

$$d\sigma = \tfrac{1}{3} Z\sigma_0 \,\mathrm{ctg}^2\, \tfrac{1}{2}\theta d(\cos\theta). \quad (12)$$

For elastic impacts with electrons one finds,[10] again using the Born approximation, an analogous result: for no spin change we again get (8); for spin change

$$d\sigma = \tfrac{2}{3}\sigma_0(1 - f + \tfrac{1}{2}f^2)df/f, \quad f < 1/(1 + \mu^2c^2/2mE). \quad (13)$$

This result is indeed of the form (4), and thus can account roughly for the size and material-dependence of large bursts. It gives

$$\kappa = 0.22.$$

This is of the order of magnitude observed, and together with (8) can probably account for the greater part of the high energy bursts.

For the bremsstrahlung, the high probability of large angle scattering given by (12), and the large coupling with high frequency radiation fields, give a cross section increasing rapidly with energy. For an unscreened Coulomb field the cross section takes the asymptotic form $\sigma \sim \alpha Z\sigma_0(E/\mu c^2)^2$; most of these are close impacts and correspond to large angles of scattering and radiation. It is therefore essential to take into account the modification of the Coulomb field within the nuclear radius $\sim Z^{\frac{1}{3}}\hbar/\mu c$. If we do this by eliminating all Fourier components of the Coulomb field with $k > \mu c/\hbar Z^{\frac{1}{3}}$, and thus ignoring nuclear collisions, we get for high E, still using the Born approximation, a cross section of the form

$$\sigma \sim \alpha Z^{\frac{2}{3}}\sigma_0 E/\mu c^2. \quad (14)$$

It is clear that neither the E nor Z dependence of this cross section is in agreement with the observations on bursts. For $E > 2\times 10^{10}$ v, (14) cannot be right. However, in the range 10^9 v $< E < 10^{10}$ v, (14) is in no contradiction to the experimental evidence: its contribution to the soft component would be only $R \sim 1$ percent.

We see, so that (13) can account roughly for soft radiation with $E > 2\times 10^{10}$ v; that (14) can surely not be right for $E > 2\times 10^{10}$ v. On what grounds can we regard (14) as invalid; and do these apply, and if so in what range of energies, to (13)?

III. Validity

Two distinct but related criteria[11] are involved in the question of the validity of both our results (13) and (14) for secondaries and bremsstrahlung: Formulae for mesotron charge and current density have been used for arbitrary small wave-lengths; one may question, as has often been done, whether they are right for distances $\ll \hbar/\mu c$. On the other hand, we have throughout treated the coupling of mesotron and electromagnetic field as small, have used, that is, a Born approximation: under what conditions is this treatment justified? The formulae (13) and (14) will surely have no validity when it is not.

The former condition is not in Lorentz-invariant form, nor can it be reformulated to apply only to the rest system of the mesotron, since in the impact the mesotron momenta change. A natural extension of the criterion is that only for those impacts in which momentum transfer π and energy transfer ϵ satisfy

$$\pi^2 - \epsilon^2/c^2 \gg \mu^2c^2 \quad (15)$$

can charge and current expressions of mesotron theory be used. This condition is thus clearly equivalent to the rejection of relativistic mesotron theory. It is incisive; there is no evidence that it is right.

To apply this condition we note that in the rest system of the center of mass of electron and mesotron the energy of the electron does not change, and the transverse and longitudinal momentum changes are of the same order of magnitude. Since the transverse momentum transfer is invariant, and is of the order $(2mE)^{\frac{1}{2}}$, we get from (15)

$$E \gg \mu^2c^2/2m \sim 10^{10} \text{ v}.$$

It is, as we have seen, only above this energy that the terms (13) become of importance compared to (8).

In the same way we may apply (15) to the bremsstrahlung. If the Coulomb field were not

[10] This result has been independently derived by Corben and Massey, Proc. Phil. Camb. Soc. in press. We are indebted to Dr. Corben for telling us of his results.

[11] W. Heisenberg, Zeits. f. Physik **110**, 251 (1938).

cut off at the nuclear radius, the scattering and radiation would be at large angles in the nuclear rest system, and (15) would give

$$E^2 \gg \mu^2 c^4.$$

For the screened field the scattering and radiation is roughly isotropic in a coordinate system S where the mesotron momentum is $(2\mu E)^{\frac{1}{2}}$, so that $\mu E \gg \mu^2 c^2$; in either case the bremsstrahlung formulae would have no range of validity at all, since they are for the case $E \gg \mu c^2$, and (15) denies that mesotron theory is applicable to such a problem.

For nuclear problems, where the relative magnitude of the coupling energy compared to the kinetic energy of the mesotrons is measured by $gE/\mu c^2$, with g a constant of order unity, the smallness of the interaction energy essentially requires (15). For electromagnetic effects the relative magnitude of the coupling is $\alpha^{\frac{1}{2}}E/\mu c^2$, $\alpha = e^2/\hbar c$; and the coupling may remain small even when (15) is violated.

It is clear from (13) and (14) that for high enough energies the coupling terms are not small. If we use for each problem the coordinate systems in which scattering is roughly isotropic, in which therefore for secondaries and bremsstrahlung, respectively, the mesotron momentum P is $(2mE)^{\frac{1}{2}}$ and $(2\mu E)^{\frac{1}{2}}$, the validity of the Born approximation at least requires that the cross section be small compared to the summed areas of the partial waves involved. For the secondaries, with nearly spherical scattering, this means that $\sigma < (\hbar/P)^2$, and thus that

$$E < \mu^2 c^2/mZ\alpha^2 \sim 4\times 10^{14}/Z \text{ v}. \qquad (16)$$

For the bremsstrahlung, the half-width, $\delta\theta$, of the angular distribution of the scattered mesotron, for fixed direction of emission of the γ-ray, is of the order $(\delta\theta)^2 \sim (\mu c/Z^{\frac{1}{3}})^2/\mu E \sim \mu c^2/Z^{\frac{2}{3}}E$; thus partial waves up to $L^2 \sim Z^{\frac{2}{3}}E/\mu c^2$ are involved; we must have $\sigma < (\hbar L/P)^2$ and

$$E < \mu c^2/\alpha^3 Z \sim 3\times 10^{14}/Z \text{ v}. \qquad (17)$$

To see whether these conditions, which are certainly *necessary* for the smallness of the coupling, are also sufficient, let us look more closely at the coupling energy density and the kinetic energy density. This is simplest in the coordinate systems S, where one may use wave packets of approximate dimensions $\hbar/P$ throughout. The kinetic energy density is then of the order $(P/\hbar)^3 Pc$ whereas the coupling energy, for those transitions involving a change of mesotron spin, is $\sim (P/\hbar)^3 \alpha^{\frac{1}{2}}(P/\mu c)Pc$, and the ratio is of the order

$$\alpha^{\frac{1}{2}}P/\mu c. \qquad (18)$$

Then we must have

$$E < \mu^2 c^2/m\alpha \sim 2\times 10^{12} \text{ v for the validity of (13),}$$

and

$$E < \mu c^2/\alpha \sim 10^{10} \text{ v for the validity of (14).}$$

These conditions are more incisive than (16,17), because the coupling of the mesotron with its radiation field becomes large before the coupling with the weaker fields of electron and nucleus do. Thus in the problem of the bremsstrahlung the maximum value of the electric field of the nucleus (in coordinate system S) acting on the mesotron is $Ze(Z^{\frac{1}{3}}\hbar/\mu c)^{-2}(E\mu/\mu c)^{\frac{1}{2}} = Z^{\frac{1}{3}}eE^{\frac{1}{2}}\mu^{\frac{3}{2}}c/\hbar^2$, whereas the zero point electric field fluctuations of wave-length $\hbar/P$, which induce mesotron radiation of this wave-length, are of the order $(\hbar c)^{\frac{1}{2}}P^2/\hbar^2 = (\hbar c)^{\frac{1}{2}}\mu E/\hbar^2$, and thus $(E/\mu c^2 \alpha Z^{\frac{2}{3}})^{\frac{1}{2}}$ larger.

The derivation of the results (13) and (14) can certainly then not be justified for energies greater than 10^{10} v for the bremsstrahlung, 2×10^{12} v for the secondaries. For to extend these results to higher energies would involve *not* treating the coupling of mesotrons and radiation as small. It is well known that this program leads to divergent results, and that only the roughest correspondence-theoretic analogies offer any guide to their interpretation. It should be emphasized that it would be of no use to treat more strictly, say in the problem of bremsstrahlung, the motion of the mesotron in the Coulomb field; it is not primarily the magnitude of *this* coupling which makes the trouble.

This discussion therefore suggests that the bremsstrahlung formula (14) may well be right up to 10^{10} v; at much higher energies the observational material on burst frequency shows that it must be wrong. On the other hand (13), for high energy secondaries, can be justified up to 2×10^{12} v, and, as we have seen, can explain at least a good part of the bursts observed.

It may be mentioned that for the inverse problem of mesotron production, pair production in a "cut-off" Coulomb field again gives a cross section of the form $\sigma=\alpha Z^{\frac{4}{3}}\sigma_0 E/\mu c^2$, and again a limit $\sim 10^{10}$ v for its validity. At this highest energy the cross section per atom of oxygen is 10^{-28} cm^2, which is about 300 times too small to account for the observed high intensity of secondary mesotrons of this energy. A cross section 300 times larger would, as a matter of fact, be inconsistent with the high penetrability of mesotrons. The large observed discrepancy between mesotron production and absorption probabilities has been repeatedly emphasized.[12]

[12] See for example, L. W. Nordheim, Phys. Rev. 56, 502 (1939).

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The Proton-Deuteron Transformation As a Source of Energy in Dense Stars*

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The rates of energy evolution due to the transformation to helium, starting with the reaction $H+H=D+e^+$, in hydrogen at densities of 10^4 to 10^8 g/cm^3, were calculated on the basis of complete degeneracy, and the assumption of a crystal-like spacing of the protons. The results indicate that any considerable amount of hydrogen in white dwarf stars would lead to much higher luminosities than those observed. Thus the low effective molecular weight (1.5) as calculated for some of these stars from the accepted white dwarf model, cannot be due to a high content of hydrogen. It might be explained as due to very large content ($\sim$100 percent) of the helium isotope He^3 but it is very difficult to see how such large amounts of this isotope could be present in these stars. It appears that the paradox can be removed only by revision of the observational data concerning the white dwarf radii.

1. Introduction

The importance of the reaction $H+H=D+e^+$ as a major source of stellar energy has been established by Bethe and Critchfield,[1] at least for stars lighter than the sun. In view of prevailing theories of stellar composition, which indicate a considerable proportion of hydrogen in most stars, it is desirable to investigate the rate of energy production due to this $H-H$ reaction in dense stars.

Since the zero-point energy of matter at high stellar densities ($\rho=10^6$ or 10^7 g/cm^3) approaches that due to stellar temperatures (10^6–10^7 °C), it is permissible, for a conservative approximation, to neglect temperatures in computing the rate of combination of protons. The results may then be interpreted as indicating a *minimum* value for the rate of energy production for assumed compositions, or as indicating a maximum hydrogen content for a given rate of energy production.

We will first consider an electron-proton gas, corresponding to a pure hydrogen composition and consider later the modifications necessary to take account of heavier particles. We assume closest cubic packing of the protons. Then, if $2r$ is the distance between nearest protons, the volume per proton is

$$4(\sqrt{2})r^3 \quad \text{and} \quad 2r=(\sqrt{2}M/\rho_H)^{\frac{1}{3}}, \tag{1}$$

where M is the mass of the proton (1.66×10^{-24} g) and ρ_H is the density of protons in g/cm^3.

2. The Potential Function

In deducing the form of the potential function governing the relative motion of two protons, the influence of neighboring protons will be considered first. The electrical potential at the equilibrium points may be taken as zero. Each proton, in its equilibrium position, is more or

* The preliminary results of this work were referred to by G. Gamow, Phys. Rev. 55, 723 (1939).

[1] H. Bethe and C. Critchfield, Phys. Rev. 54, 248 (1938).